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# AS Level Mathematics A

## H230/02 Pure Mathematics and Mechanics

Sample Question Paper

**Date – Morning/Afternoon** Version 2

Time allowed: 1 hour 30 minutes

**You must have:**
- Printed Answer Booklet

**You may use:**
- a scientific or graphical calculator

Model Answers

## INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

## INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

**Formulae**
**AS Level Mathematics A (H230)**

**Binomial series**

$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \dots\ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \dots\ + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

**Differentiation from first principles**

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

**Standard deviation**

$$\sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \ \text{ or } \ \sqrt{\frac{\Sigma f\left(x-\bar{x}\right)^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$$

**The binomial distribution**

If $X \sim \mathrm{B}(n,\ p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of $X$ is $np$, variance of $X$ is $np(1-p)$

**Kinematics**

$v = u + at$

$s = ut + \frac{1}{2}at^2$

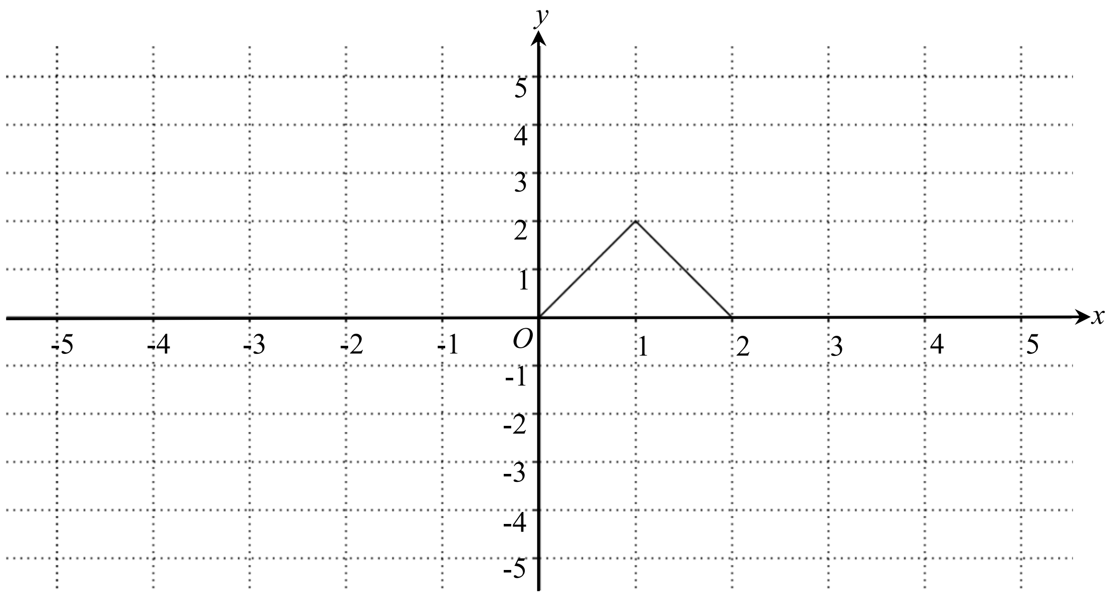
$s = \frac{1}{2}\left(u + v\right)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

**Section A: Pure Mathematics**
Answer **all** the questions

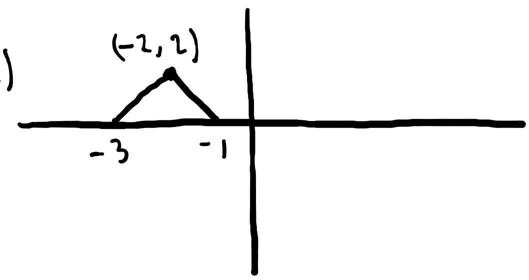
**1** **(a)** The diagram below shows the graph of .




**(i)** On the diagram in the Printed Answer Booklet draw the graph of $y = \mathrm{f}(x+3)$. **[2]**

1

a) i)




**(ii)** Describe fully the transformation which transforms the graph of $y = \mathrm{f}(x)$ to the graph of $y = -\mathrm{f}(x)$. **[1]**

ii) The graph of $y = F(x)$ is reflected in the $x$-axis to get $y = -F(x)$.

**(b)** The point (2, 3) lies on the graph of $y = g(x)$.

State the coordinates of its image when $y = g(x)$ is transformed to

**(i)** $y = 4g(x)$ **[1]**

b)i) $y = 4g(x)$ $y = aF(x)$

$(2,3) \rightarrow (2,12)$

**(ii)** $y = g(4x)$. **[1]**

ii) $y = g(4x)$ $y = F(ax)$

Scale Factor $= \frac{1}{a}$

$(2,3) \rightarrow (1/2, 3)$

**2** **In this question you must show detailed reasoning.**

Solve the equation $2\cos^2 x = 2 - \sin x$ for $0^\circ \le x \le 180^\circ$. **[5]**

2

$2\cos^2 x = 2 - \sin x$

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$2(1 - \sin^2 x) = 2 - \sin x$

$2 - 2\sin^2 x = 2 - \sin x$

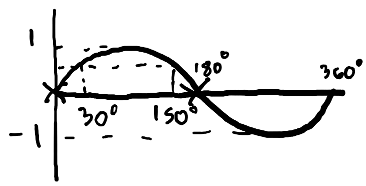
$2 = 2 + 2\sin^2 x - \sin x$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0$ $2\sin x - 1 = 0$

$\sin x = 1/2$




$x = \sin^{-1}(0) = 0$

$x = 0°$ or $x = 180°$

$x = \sin^{-1}(1/2) = 30$

$x = 30°$ or $x = 150°$

$x = 0°, 30°, 150°, 180°$

**3** The number of members of a social networking site is modelled by $m = 150e^{2t}$, where $m$ is the number of members and $t$ is time in weeks after the launch of the site.

**(a)** State what this model implies about the relationship between $m$ and the rate of change of $m$. **[2]**

a) $m = 150e^{2t}$ $\frac{dm}{dt} = 300e^{2t} = 2m$

Rate of change of the members over a certain number of weeks is proportional to twice the number of members.

**(b)** What is the significance of the integer 150 in the model? **[1]**

b) When $t=0$, 150 is the number of members at the start of the launch for the social networkin site.

**(c)** Find the week in which the model predicts that the number of members first exceeds 60 000. **[3]**

c) $m = 150e^{2t}$

$150e^{2t} > 60{,}000$

$e^{2t} > 400$

$\ln e^{2t} > \ln 400$

$2t > \ln 400$

$t > \frac{\ln 400}{2}$

$t > 2.9957322\ldots$

$t = 3$ weeks

t must be the third week in which the number of members exceeds 60,000.

**(d)** The social networking site only expects to attract 60 000 members.

Suggest how the model could be refined to take account of this. **[1]**

d) The time taken should only be restricted to 3 weeks.

$(0 \leq t \leq 3)$

**4** The points $A$, $B$ and $C$ have position vectors $\begin{pmatrix}-2\\1\end{pmatrix}$, $\begin{pmatrix}2\\5\end{pmatrix}$ and $\begin{pmatrix}6\\3\end{pmatrix}$ respectively.

$M$ is the midpoint of $BC$.

**(a)** Find the position vector of the point $D$ such that $\overrightarrow{BC} = \overrightarrow{AD}$. **[3]**

a) $\overrightarrow{BC} = \begin{pmatrix}6\\3\end{pmatrix} - \begin{pmatrix}2\\5\end{pmatrix} = \begin{pmatrix}4\\-2\end{pmatrix}$

$\overrightarrow{AD} = \begin{pmatrix}4\\-2\end{pmatrix}$ $\quad D = \begin{pmatrix}x\\y\end{pmatrix}$

$\overrightarrow{AD} = \begin{pmatrix}x\\y\end{pmatrix} - \begin{pmatrix}-2\\1\end{pmatrix} = \begin{pmatrix}x+2\\y-1\end{pmatrix} = \begin{pmatrix}4\\-2\end{pmatrix}$

$\begin{pmatrix}x+2\\y-1\end{pmatrix} = \begin{pmatrix}4\\-2\end{pmatrix}$

$x+2=4$ $\quad y-1=-2$

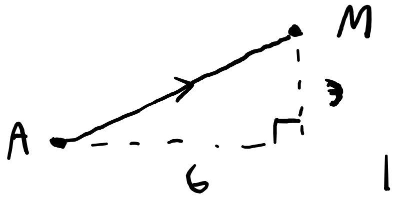
$x=2$ $\quad y=-1$

$D = \begin{pmatrix}2\\-1\end{pmatrix}$

**(b)** Find the magnitude of $AM$. **[3]**

b) $M = \begin{pmatrix}\left(\frac{2+6}{2}\right)\\\left(\frac{5+3}{2}\right)\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix}$

$\overrightarrow{AM} = \begin{pmatrix}4\\4\end{pmatrix} - \begin{pmatrix}-2\\1\end{pmatrix} = \begin{pmatrix}6\\3\end{pmatrix}$




$|\overrightarrow{AM}| = \sqrt{6^2+3^2} = \sqrt{45}$

$|\overrightarrow{AM}| = 3\sqrt{5}$

**5** A doctors' surgery starts a campaign to reduce missed appointments.
The number of missed appointments for each of the first five weeks after the start of the campaign is shown below.

| Number of weeks after the start ($x$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of missed appointments ($y$) | 235 | 149 | 99 | 59 | 38 |

This data could be modelled by an equation of the form $y = pq^x$ where $p$ and $q$ are constants.

**(a)** Show that this relationship may be expressed in the form $\log_{10} y = mx + c$, expressing $m$ and $c$ in terms of $p$ and/or $q$. **[2]**

a) $y = pq^c$

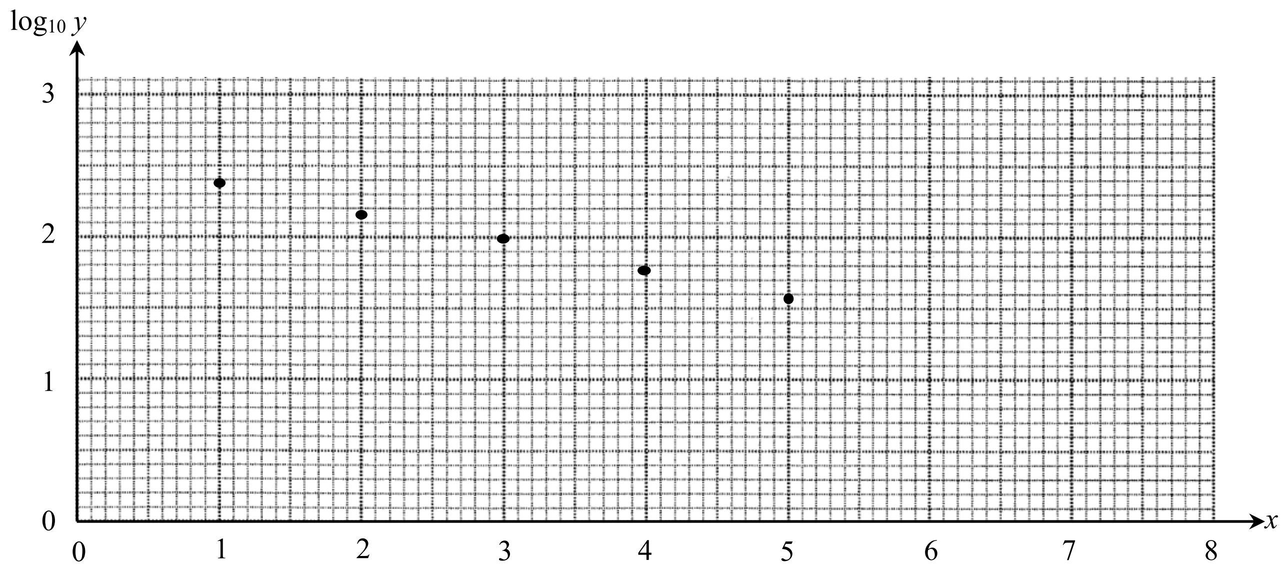
$\log_{10} y = \log_{10} pq^x$

$\log_{10} y = \log_{10} p + \log_{10} q^x$

$\log_{10} y = \log_{10} p + x \log_{10} q$

$m = \log_{10} q \quad c = \log_{10} p$

The diagram below shows $\log_{10} y$ plotted against $x$, for the given data.



**(b)** Estimate the values of $p$ and $q$. **[3]**

b) Draw on graph straight line through the plotted points

**[2]**

$\log_{10} y = \log_{10} p + x \log_{10} q$

$y = mx + c$

$m = \log_{10} q \qquad c = \log_{10} p$

Draw a gradient triangle between 2 points:

$(1, 2.4) \qquad (5, 1.6)$

$\log_{10} q = \frac{dy}{dx} = \frac{2.4 - 1.6}{1 - 5} = -0.2$

$\log_{10} q = -0.2$

$q = 10^{-0.2} = 0.631$ (3 sF)

$-2.5 - \log_{10} p$

$p = 10^{2.5} = 316$ (3sF)

**(c)** Use the model to predict when the number of missed appointments will fall below 20.

Explain why this answer may not be reliable. **[2]**

c) When $y = 20$ $\log_{10} 20 = 1.30$ (3sF)

Draw straight horizontal line at 1.30. No. of missed appointments will fall below 20 around the $7^{th}$ week.

The model is not reliable because we are assuming that the model will hold true over a long period of time.

6 **(a)** A student suggests that, for any prime number between 20 and 40, when its digits are squared and then added, the sum is an odd number.

For example, 23 has digits 2 and 3 which gives $2^2 + 3^2 = 13$, which is odd.

Show by counter example that this suggestion is false. **[2]**

a) 29 $2^2 + 9^2 = 4 + 81 = 85$ (odd)

31 $3^2 + 1^2 = 9 + 1 = 10$ (even)

This is the disproof counter-example

**(b)** Prove that the sum of the squares of any three consecutive positive integers cannot be divided by 3. **[3]**

b) $n$, $n+1$, $n+2$

$n^2 + (n+1)^2 + (n+2)^2$

$= n^2 + (n+1)(n+1) + (n+2)(n+2)$

$= n^2 + n^2 + 2n + 1 + n^2 + 2n + 2n + 4$

$= 3n^2 + 6n + 5 = 3n(n+2) + 5$

$3n(n+2) + 5$ has a remainder of 5 $\therefore$ Cannot be divided by 3

**7** Differentiate $f(x) = x^4$ from first principles. **[5]**

7 $F(x) = x^4$ $F'(x) = 4x^3$

$$f'(x) = \frac{(x+h)^4 - x^4}{h} \quad h \to 0$$

$$= \frac{(x+h)^2(x+h)^2 - x^4}{h} = \frac{(x^2+2x+h^2)^2 - x^4}{h}$$

$$= \frac{(x^2+2xh+h^2)(x^2+2xh+h^2) - x^4}{h}$$

$$= \frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4 - x^4}{h}$$

$$\lim_{h \to 0} \frac{4x^3h+6x^2h^2+4xh^3+h^4}{h}$$

$$= \lim_{h \to 0} 4x^3+6x^2h+4xh^2+h^3$$

$$= 4x^3$$

**8** A curve has equation $y \quad kx^{\frac{3}{2}}$ where $k$ is a constant.
The point $P$ on the curve has $x$-coordinate 4.
The normal to the curve at $P$ is parallel to the line $2x+3y=0$ and meets the $x$-axis at the point $Q$.
The line $PQ$ is the radius of a circle centre $P$.

Show that $k=\frac{1}{2}$.
Find the equation of the circle. **[10]**

8 $P(4, 8k)$ $\quad y = k(4)^{3/2}$
$= 8k$

$2x+3y=0$
$3y=-2x$
$y=-\frac{2}{3}x$

Gradient of normal at $P = -\frac{2}{3}$

Gradient of tangent at $P = \frac{3}{2}$

$y = kx^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}kx^{1/2}$

$\frac{3}{2} = \frac{3}{2}kx^{1/2}$

$1 = kx^{1/2}$ $\quad$ At $P$, $x=4$

$1 = k(4)^{1/2}$

$k = \frac{1}{2}$

$P(4,4)$

$y - y_1 = m(x-x_1)$ $\quad \frac{dy}{dx}_{PQ} = -\frac{2}{3}$

$y-4 = -\frac{2}{3}(x-4)$

$y-4 = -\frac{2}{3}x + \frac{8}{3}$

$y = -\frac{2}{3}x + \frac{20}{3}$

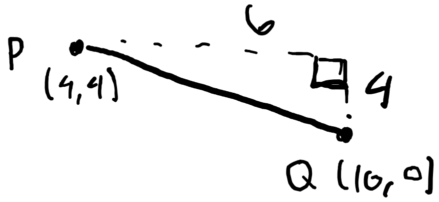
When $y=0$

$$0 = -\tfrac{2}{3}x + \tfrac{20}{3}$$

$$\tfrac{2}{3}x = \tfrac{20}{3}$$

$$x = 10$$

$Q(10, 0)$




$$PQ = \sqrt{6^2+4^2} = \sqrt{52} = 2\sqrt{13}$$

$$(x-a)^2 + (y-b^2) = r^2$$

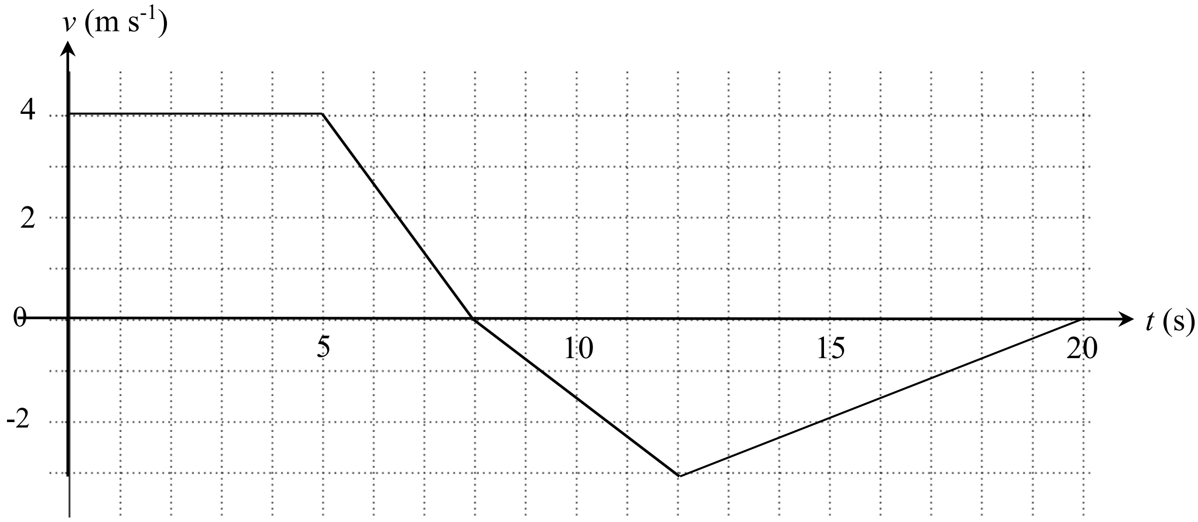
$(a, b)$ is centre of circle

$$(x-4)^2 + (y-4)^2 = (2\sqrt{13})^2$$

$$(x-4)^2 + (y-4)^2 = 52$$

**Section B: Mechanics**
Answer **all** the questions

**9** The diagram below shows the velocity-time graph of a car moving along a straight road, where $v$ m s$^{-1}$ is the velocity of the car at time $t$ s after it passes through the point $A$.




**(a)** Calculate the acceleration of the car at . **[2]**

a) $a = \frac{-4}{8-5} = -\frac{4}{3}$

acceleration at $t=6$ is $-\frac{4}{3}$ $ms^{-2}$

**(b)** Jasmit says "The distance travelled by the car during the first 20 seconds of the car's motion is more than five times its displacement from $A$ after the first 20 seconds of the car's motion".

Give evidence to support Jasmit's statement. **[3]**

b) distance = speed × time

distance $= (5\times4) + (\frac{1}{2}\times3\times4) + (\frac{1}{2}\times4\times3) + (\frac{1}{2}\times8\times3)$

$= 44$ m

displacement $= (5\times4) + (\frac{1}{2}\times3\times4) - (\frac{1}{2}\times4\times3) - (\frac{1}{2}\times8\times3)$
$= 8$ m

$44 > 5\times8$

**10** A student is attempting to model the flight of a boomerang.
She throws the boomerang from a fixed point $O$ and catches it when it returns to $O$.

She suggests the model for the displacement, $s$ metres, after $t$ seconds is given by

$s = 9t^2 - \frac{3}{2}t^3,\ 0 \le t \le 6.$

For this model,

**(a)** determine what happens at $t = 6$, **[2]**

a) $t = 6$

$s = 9(6)^2 - \frac{3}{2}(6)^3 = 0$

When $t=6$, this is the point at which the student catches the boomerang.

**(b)** find the greatest displacement of the boomerang from $O$, **[4]**

b) When $v=0$

$s = 9t^2 - \frac{3}{2}t^3$

$\frac{ds}{dt} = v = 18t - \frac{9}{2}t^2$

$0 = 18t - \frac{9}{2}t^2$
$0 = t(-\frac{9}{2}t + 18)$

$t = 0$ $\quad$ $18 = \frac{9}{2}t$
$t = 4$

When $t=4$ $\quad$ $s = 9(4)^2 - \frac{3}{2}(4)^3 = 48m$

**(c)** find the velocity of the boomerang 1 second before the student catches it, **[2]**

c) $v = 18t - \frac{9}{2}t^2$ when $t = 5$

$v = 18(5) - \frac{9}{2}(5)^2 = -22.5\ ms^{-1}$

**(d)** find the acceleration of the boomerang 1 second before the student catches it. **[2]**

d) $v = 18t - \frac{9}{2}t^2$

$\frac{dv}{dt} = 18 - 9t = a$ when $t = 5$

$a = 18 - 9(5) = -27\ ms^{-2}$

**11** In this question the unit vectors **i** and **j** are in the directions east and north respectively.

Distance is measured in metres and time in seconds.

A ship of mass 100 000 kg is being towed by two tug boats.

- The cables attaching each tug to the ship are horizontal.
- One tug produces a force of $(350\mathbf{i} + 400\mathbf{j})$ N.
- The other tug produces a force of $(250\mathbf{i} - 400\mathbf{j})$ N.
- The total resistance to motion is 200 N.
- At the instant when the tugs begin to tow the ship, it is moving east at a speed of 1.5 m $s^{-t}$.

**(a)** Explain why the ship continues to move directly east. **[2]**

a) Resultant forces in i direction

$350 + 250 - 200 = 400N$ (East)

Resultant forces in j direction

$400 - 400 = 0N$

There is 0 resultant force acting in the j direction so neither of the tug boats are moving North or South. Therefore overall resultant force acts 400N due east

**(b)** Find the acceleration of the ship. **[2]**

b) $F = ma$

$400 = 100000a$

$a = 0.004\ ms^{-2}$

**(c)** Find the time which the ship takes to move 400 m while it is being towed. Find its speed after moving that distance. **[6]**

c) $s = 400$
$u = 1.5$
$v = X$
$a = 0.004$
$t = ?$

$s = ut + \frac{1}{2}at^2$
$400 = 1.5t + 0.002t^2$

$0.002t^2 + 1.5t - 400 = 0$

use quadratic equation

$t = 209\,s \quad t = -959\,s$ X

$s = 400$
$u = 1.5$
$v = ?$
$a = 0.004$
$t =$

$v^2 = u^2 + 2as$
$v^2 = 1.5^2 + 2(0.004)(400)$

$v^2 = 5.45$

$v = 2.33\ ms^{-1}$